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Zip bifurcation in an ample class of competitive systems

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ZIP BIFURCATION IN AN AMPLE CLASS OF COMPETITIVE SYSTEMS

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Abstract. In this research we study the occurrence of *zip bifurcation* in an ample class of $(n + 1)$ -dimensional prey–predator system modeling the competition among n species of predators for one species of prey. A similar study was made first by Farkas [3] for a three dimensional prey–predator system where he studied the competition between two species of predators for one species of prey.

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1. INTRODUCTION

The phenomenon of a *zip bifurcation* had been discovered by Farkas [3] in 1984 for a three-dimensional prey-predator system. The model was not a structurally stable one, however, it illustrated the intuitively evident fact that at low values of the carrying capacity K both predators could survive but as K grew, only one of them survived. Recently, theoretical results related to this phenomenon have been generalized first to a four-dimensional system [1, 4] and later to an n -dimensional ordinary differential system [5]. The first model [1] arose in economy and politology, whereas the other one [4, 5] appeared in a specific prey-predator model.

The purpose of this paper is to study the phenomenon of *zip bifurcation* in an ample class of ordinary differential systems modeling the competition among n species of predators for a single prey. The model under consideration is described by the system of ordinary differential equations

$$\begin{aligned}\dot{S} &= \gamma g(S, K)S - \sum_{i=1}^n p(S, a_i)x_i, \\ \dot{x}_i &= p(S, a_i)x_i - d_i x_i, \quad i = 1, \dots, n.\end{aligned}\tag{1.1}$$

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The parameters involved in system (1.1) are all non-negative and have the following meaning:

| | |
|------------------|---|
| S | quantity of prey |
| x_i | quantity of predator i |
| $\gamma g(S, K)$ | per capita growth rate of prey in absence of predators* |
| K | carrying capacity of the environment with respect to the prey |
| d_i | death rate of the i th predator |
| $p(S, a_i)$ | per capita birth rate of the i th predator, where a_i is a constant |

The function $g(S, K)$ possesses the following properties:

$$g \in C^2([0, \infty[\times]0, \infty[), \quad g \in C^0([0, \infty[\times]0, \infty[), \\ g(0, K) = 1, \quad g'_S(S, K) < 0 < g''_{SK}(S, K), \quad S \geq 0, K > 0. \quad (1.2)$$

Condition (1.2) means that the highest specific growth rate of prey is achieved at $S = 0$, $x_1 = 0$, $x_2 = 0$, and it is $\gamma > 0$; the growth rate decreases if the quantity of prey increases, and the rate of decrease in growth rate $g'_S(S, K)$ is negative and an increasing function of the carrying capacity K , i. e., the effect of the increase in prey diminishes with an increase in K .

$$\lim_{K \rightarrow \infty} g'_S(S, K) = 0 \quad (1.3)$$

uniformly in $[\delta, S_0[$ for any $0 < \delta < S_0$, and the improper integral $\int_0^{S_0} g'_S(S, K) dS = 0$ is uniformly convergent in $[K_0, \infty[$ for any $K_0 > 0$. Relation (1.3) means that for very high values of K changes in the quantity of prey have a negligible effect on the growth rate. It is easy to see that conditions (1.2)–(1.3) imply that

$$\lim_{K \rightarrow \infty} g(S, K) = 1, \quad S \geq 0 \quad (1.4)$$

and

$$(K - S)g(S, K) > 0, \quad S \geq 0, K > 0, S \neq K. \quad (1.5)$$

Inequality (1.4) means that (in absence of predator) the growth rate of prey $g(S, K) > 0$ if $S < K$ and $g(S, K) < 0$ if $S > K$ therefore, by continuity of function g we have $g(K, K) = 0$.

The function $p(S, a)$ possesses the following properties:

$$p \in C^2([0, \infty[\times]0, \infty[), \quad p \in C^0([0, \infty[\times]0, \infty[), \\ p(0, a) = 0, \quad p'_S(S, a) > 0, \quad S > 0, a > 0 \quad (1.6)$$

Conditions (1.6) mean that the per capita birth rate p of the predators (also called the “predation rate” or the “functional response”) is zero in the absence of prey and is an increasing function of the quantity of prey.

$$p'_S(S, a) < \frac{p(S, a)}{S}, \quad S > 0, a > 0. \quad (1.7)$$

Condition (1.7) is a “weak concavity” condition, sometimes called Krassonselskij’s condition. If p is a strictly concave function of S (for any $a > 0$), (1.7) is implied with the possible exception of isolated points where it holds with an equality sign.

$$p'_a(S, a) < 0, \quad S > 0, \quad a > 0. \quad (1.8)$$

Inequality (1.8) throws light on the role of the parameter a ; the birth rate of the predator is a decreasing function of a , i. e., the higher the value of a is the more food is needed to maintain the same birth rate of the specific predator. In the original model of Hsu, Hubbell, and Waltman [7] a is the “half-saturation constant”. The conditions imply that $p(S, a)$, S and a are all greater than zero. In the case where p is a bounded function for fixed $a > 0$, $m_i = \sup_{S>0} p(S, a_i)$ is the “maximal birth rate” of the i th predator. Clearly,

$$\lim_{S \rightarrow \infty} p(S, a_i) = \begin{cases} m_i & \text{if } p \text{ is bounded,} \\ 0 & \text{if } p \text{ is not bounded.} \end{cases} \quad (1.9)$$

We shall assume that $a_1 > a_2 > \dots > a_n$, i. e.,

$$p(S, a_i) < p(S, a_{i+1}) \quad \text{for all } S > 0 \quad (1.10)$$

according to condition (1.8). We shall also assume from now on that $d_1 < d_2 < \dots < d_n$, as a consequence, (1.10) does not imply that the net growth rate of predator $i + 1$ also exceeds that of predator i .

Another important characteristic of the respective predator species is the prey *threshold quantity* $S = \lambda_i$, above which their growth rate is positive, i. e., $p(\lambda_i, a_i) = d_i$, $i = 1, 2, \dots, n$. However, we shall assume that $\lambda_1 = \lambda_2 = \dots = \lambda_n$, i. e., the n species have equal prey thresholds although they achieve this by different means. Thus, our assumption will be that there exists a $\lambda > 0$ such that

$$p(\lambda, a_i) = d_i, \quad i = 1, 2, \dots, n. \quad (1.11)$$

We note that, because of condition (1.6), equation (1.11) has one and only one solution λ if and only if either p or m_i is greater than d_i . The real content of (1.11) is that the n solutions for $i = 1, 2, \dots, n$ coincide.

In the next section we study the equilibrium points for the system and we prove its dissipativeness. In Section 3 we establish the conditions under which zip bifurcation occurs in the model.

2. EQUILIBRIUM POINTS

We first show that system (1.1) is dissipative before studying its equilibrium points.

Proposition 1. *Any solution of system (1.1) with initial values $S^0 > 0$, $x_i^0 > 0$, $i = 1, 2, \dots, n$, is bounded in $[0, \infty[$.*

Proof. We first observe that any solution of (1.1) whose initial value has positive components remains with positive components, as long as the solution exists. We will prove that the solution exists for all time $t \geq 0$ and there exists a bounded J in \mathbb{R}_+^{n+1} which attracts the solutions starting on any bounded set in \mathbb{R}_+^{n+1} .

Let $d_0 = \min\{d_1, \dots, d_n\}$ and

$$V(S, x_1, \dots, x_n) = S + x_1 + \dots + x_n.$$

If $z(t) = (S(t), x_1(t), \dots, x_n(t))$ is a solution of (1.1), then as long as it exists, we have

$$\frac{d}{dt}V(z(t)) = \gamma g(S, K)S - \sum_{i=1}^n d_i x_i.$$

Since $\gamma g(S, K)S < -dS + 2\gamma K$, where $0 < d < \gamma$ for all $S \in \mathbb{R}_+$, according to conditions (1.2) and (1.5), we have

$$\frac{d}{dt}V(z(t)) \leq -dS - \sum_{i=1}^n d_i x_i + 2\gamma K.$$

Letting $\alpha = \min\{d_0, d\}$, we have

$$\frac{d}{dt}V(z(t)) \leq -\alpha V(z(t)) + 2\gamma K$$

and therefore

$$V(z(t)) \leq V(z(0))e^{-\alpha t} + \frac{2\gamma K}{\alpha}$$

as long as the solution exists. If B is a bounded set contained in \mathbb{R}_+^{n+1} , then there exists $R > 0$ such that $V(z(0)) \leq R$. Let $t_0 = \frac{1}{\alpha} \log \frac{2\gamma K}{\alpha R}$ and $z(0) \in B$. For $t \geq t_0$, we have

$$V(z(t)) \leq R \frac{2\gamma K}{\alpha R} + \frac{2\gamma K}{\alpha} \leq \frac{4\gamma K}{\alpha}.$$

This implies that any solution is defined for $t \geq 0$ and the compact set

$$J = \left\{ (S, x_1, \dots, x_n) : S \geq 0, x_1 \geq 0, \dots, x_n \geq 0, \right. \\ \left. \text{and } S + x_1 + \dots + x_n \leq \frac{\gamma K(1 + d_0)^2}{2\alpha} \right\}$$

attracts all bounded sets B . Therefore, the system is dissipative and its global attractor is contained in J . \square

Let us now consider the equilibrium points of (1.1), that is, the solutions of the system

$$\begin{aligned} \gamma g(S, K)S - \sum_{i=1}^n p(S, a_i)x_i &= 0, \\ (p(S, a_i) - d_i)x_i &= 0, \quad i = 1, 2, \dots, n. \end{aligned} \quad (2.1)$$

For $i = 1, \dots, n$, let λ_i be the prey *quantity threshold* for species i . Then, apart from the obvious two solutions $(S, x_1, \dots, x_n) = (0, 0, \dots, 0)$ and $(S, x_1, \dots, x_n) = (K, 0, \dots, 0)$, equation (2.1) has biologically interesting solutions only if $m_i > d_i$ and $\lambda_1 = \dots = \lambda_n$. Henceforth, we assume that $m_i > d_i$ and $\lambda_1 = \dots = \lambda_n = \lambda$, i. e., the n species have equal prey thresholds although they achieve this by different means. In order to have the predators' survival it is necessary that $m_i > d_i$, $i = 1, 2, \dots, n$. Predator i begins to grow only if the right-hand side of the i th equation in system (1.1) is positive, i. e., if

$$(p(S, a_i) - d_i)x_i > 0,$$

and it means by condition (1.6) that $S > \lambda$, where $0 < \lambda < K$.

The equilibrium points of system (1.1) are the origin $(S, x_1, \dots, x_n) = (0, 0, \dots, 0)$, the point $(S, x_1, \dots, x_n) = (K, 0, \dots, 0)$ and the points of the $(n-1)$ -dimensional hyperplane

$$\begin{aligned} H_K = \{ (S, x_1, \dots, x_n) \in \mathbb{R}^{n+1} : S = \lambda, x_i \geq 0, i = 1, \dots, n, \\ p(\lambda, a_1)x_1 + p(\lambda, a_2)x_2 + \dots + p(\lambda, a_n)x_n = \gamma\lambda g(\lambda, K) \}. \end{aligned} \quad (2.2)$$

To study the stability of these equilibrium points, observe that the Jacobian matrix $J(S, x_1, \dots, x_n)$ of system (1.1) is

$$\begin{pmatrix} \gamma g(S, K) + \gamma g'_S(S, K)S & -p(S, a_1) & -p(S, a_2) & \dots & -p(S, a_n) \\ -\sum_{i=1}^n p'_S(S, a_i)x_i & p(S, a_1) - d_1 & 0 & \dots & 0 \\ p'_S(S, a_1)x_1 & 0 & p(S, a_2) - d_2 & \dots & 0 \\ p'_S(S, a_2)x_2 & 0 & 0 & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p'_S(S, a_n)x_n & 0 & 0 & \dots & p(S, a_n) - d_n \end{pmatrix}$$

Now, we have

$$J(0, 0, \dots, 0) = \begin{pmatrix} \gamma & 0 & 0 & \dots & 0 \\ 0 & -d_1 & 0 & \dots & 0 \\ 0 & 0 & -d_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -d_n \end{pmatrix}$$

so $(S, x_1, \dots, x_n) = (0, 0, \dots, 0)$ is unstable, with a n -dimensional stable manifold and a 1-dimensional unstable manifold. On the other hand,

$$J(K, 0, \dots, 0) = \begin{pmatrix} \gamma K g'_S(K, K) & p(K, a_2) & -p(K, a_2) & \dots & -p(K, a_n) \\ 0 & p(K, a_1) - d_1 & 0 & \dots & 0 \\ 0 & 0 & p(K, a_2) - d_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & p(K, a_n) - d_n \end{pmatrix}$$

by conditions (1.6) and (1.11), $(S, x_1, \dots, x_n) = (K, 0, \dots, 0)$ is asymptotically stable if $K < \lambda$ and unstable if $K > \lambda$ with a 1-dimensional stable manifold and a n -dimensional unstable manifold. It is well-known (see [6] and [2]) that

$$K > \lambda \quad (2.3)$$

is a necessary condition for the survival of each predator. Therefore, (2.3) will also be assumed from now on. Note that, by (1.4), if $K < \lambda$, then H_K is empty, and if $K = \lambda$, then its only point is the origin. In the next section we fix our attention on the study of stability of the equilibrium points belonging to H_K .

3. COEXISTENCE AND EXTINCTION BY ZIP BIFURCATION

In this section we shall study the stability of the hyperplane H_K . The elements of H_K are denoted by $(\lambda, \xi_1, \dots, \xi_n)$, i. e., $(\lambda, \xi_1, \dots, \xi_n) \in H_K$.

Evaluating the Jacobian matrix of (1.1) at an arbitrary point of H_K we get that $J = J(\lambda, \xi_1, \dots, \xi_n)$ is given by the formula

$$\begin{pmatrix} \gamma g(\lambda, K) + \gamma \lambda g'_S(\lambda, K) & -p(\lambda, a_1) & -p(\lambda, a_2) & \dots & -p(\lambda, a_n) \\ -\sum_{i=1}^n p'_S(\lambda, a_i) \xi_i & 0 & 0 & \dots & 0 \\ p'_S(\lambda, a_1) \xi_1 & 0 & 0 & \dots & 0 \\ p'_S(\lambda, a_2) \xi_2 & 0 & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ p'_S(\lambda, a_n) \xi_n & 0 & 0 & \dots & 0 \end{pmatrix}$$

since by condition (1.11) we have $p(\lambda, a_i) - d_i = 0$, $i = 1, 2, \dots, n$. So, the characteristic polynomial is

$$P(\mu) = \left(\mu - \gamma g(\lambda, K) - \gamma \lambda g'_S(\lambda, K) + \sum_{i=1}^n p'_S(\lambda, a_i) \xi_i \right) \mu^n + p(\lambda, a_1) \Delta_{12} + p(\lambda, a_2) \Delta_{13} + \dots + p(\lambda, a_n) \Delta_{1n+1}.$$

where $\Delta_{1j} = (-1)^{j+1} \det(\mu - J)_{1j}$, and $\det(\mu - J)_{1j}$ is the determinant of the sub-matrix $(\mu - J)_{1j}$ obtained from $\mu - J$ by eliminating line 1 and column j , $j = 2, 3, \dots, n+1$, i. e., of the matrix

$$\begin{pmatrix} -p'_S(\lambda, a_1)\xi_1 & \mu & 0 & 0 & \dots & \overset{\text{col}(j-1)}{0} & \overset{\text{col}(j+1)}{0} & \dots & 0 & 0 \\ -p'_S(\lambda, a_2)\xi_2 & 0 & \mu & 0 & \dots & 0 & 0 & \dots & 0 & 0 \\ -p'_S(\lambda, a_3)\xi_3 & 0 & 0 & \mu & \dots & 0 & 0 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \dots & \vdots & \vdots \\ -p'_S(\lambda, a_{n-1})\xi_{n-1} & 0 & 0 & 0 & \dots & 0 & 0 & \dots & \mu & 0 \\ -p'_S(\lambda, a_n)\xi_n & 0 & 0 & 0 & \dots & 0 & 0 & \dots & 0 & \mu \end{pmatrix}.$$

So we have

$$\begin{aligned} \Delta_{12} &= (-1)^3 \left((-1)^2 (-p'_S(\lambda, a_1)) \mu^{n-1} \right) = \xi_1 p'_S(\lambda, a_1) \mu^{n-1}, \\ \Delta_{13} &= \xi_2 p'_S(\lambda, a_2) \mu^{n-1}, \\ &\vdots \\ \Delta_{1n} &= \xi_n p'_S(\lambda, a_n) \mu^{n-1}. \end{aligned}$$

Thus, the characteristic polynomial associated to $J(\lambda, \xi_1, \dots, \xi_n)$ is

$$\begin{aligned} P(\mu) &= \left(\mu - \gamma g(\lambda, K) - \gamma \lambda g'_S(\lambda, K) + \sum_{i=1}^n p'_S(\lambda, a_i) \xi_i \right) \mu^n \\ &\quad + \xi_1 p(\lambda, a_1) p'_S(\lambda, a_1) \mu^{n-1} + \xi_2 p(\lambda, a_2) p'_S(\lambda, a_2) \mu^{n-1} \\ &\quad + \dots + \xi_n p(\lambda, a_n) p'_S(\lambda, a_n) \mu^{n-1} \\ &= \mu^{n-1} \left[\mu^2 + \left(\sum_{i=1}^n p'_S(\lambda, a_i) \xi_i - \gamma g(\lambda, K) - \gamma \lambda g'_S(\lambda, K) \right) \mu \right. \\ &\quad \left. + \sum_{i=1}^n \xi_i p(\lambda, a_i) p'_S(\lambda, a_i) \right]. \end{aligned} \tag{3.1}$$

This means that each equilibrium point in H has 0 as an eigenvalue with multiplicity $n-2$ and two eigenvalues with real part negative if the quadratic polynomial in square brackets is stable, respectively positive if the polynomial is unstable. Now, the quadratic polynomial in square brackets is stable if and only if

$$\sum_{i=1}^n p'_S(\lambda, a_i) \xi_i > \gamma g(\lambda, K) + \gamma \lambda g'_S(\lambda, K).$$

We can rewrite the above inequality as follows

$$\gamma\lambda g(\lambda, K) + \gamma\lambda^2 g'_S(\lambda, K) < \sum_{i=1}^n (\lambda p'_S(\lambda, a_i) - p(\lambda, a_i)) \xi_i + \sum_{i=1}^n p(\lambda, a_i) \xi_i.$$

Now, $(\xi_1, \xi_2, \dots, \xi_n)$ satisfy (2.2), i. e., $\sum_{i=1}^n p(\lambda, a_i) \xi_i = \gamma\lambda g(\lambda, K)$, hence we obtain

$$\sum_{i=1}^n (p(\lambda, a_i) - \lambda p'_S(\lambda, a_i)) \xi_i < -\gamma\lambda^2 g'_S(\lambda, K). \quad (3.2)$$

By condition (1.7) the left-hand side of (3.2) is positive for all $(\xi_1, \xi_2, \dots, \xi_n) \in H_K$. In view of (1.2) and (1.3) the right-hand side is positive, decreases and tends to zero for $K \rightarrow \infty$. Let us consider the hyperplane B_K given by

$$B_K = \left\{ (S, x_1, \dots, x_n) \in \mathbb{R}^{n+1} : S = \lambda, \xi_i \geq 0, i = 1, \dots, n, \right. \\ \left. \sum_{i=1}^n [p(\lambda, a_i) - \lambda p'_S(\lambda, a_i)] \xi_i = -\gamma\lambda^2 g'_S(\lambda, K) \right\}. \quad (3.3)$$

Let us denote the i th intersection coordinate of H_K with the coordinate axes by $(0, \dots, 0, x_i^{H_K}, 0, \dots, 0)$ and of B_K by $(0, \dots, 0, x_i^{B_K}, 0, \dots, 0)$, where

$$x_i^{H_K} = \frac{\gamma\lambda g(\lambda, K)}{p(\lambda, a_i)}, \\ x_i^{B_K} = -\frac{\gamma\lambda^2 g'_S(\lambda, K)}{p(\lambda, a_i) - \lambda p'_S(\lambda, a_i)}, \quad i = 1, \dots, n. \quad (3.4)$$

Observe that by conditions (1.6) and (1.9) for $K = \lambda$ we have

$$x_i^{H_\lambda} = 0 \quad \text{and} \quad x_i^{B_\lambda} > 0 \quad \text{for all} \quad i = 1, \dots, n. \quad (3.5)$$

On the other hand, $\lim_{K \rightarrow \infty} g'_S(S, K) = 0$ and $\lim_{K \rightarrow \infty} g(S, K) = 1$, so by continuity there exist $K = K_i > \lambda$ such that $x_i^{H_{K_i}} = x_i^{B_{K_i}}$, i. e.,

$$\frac{\gamma\lambda g(\lambda, K)}{p(\lambda, a_i)} = -\frac{\gamma\lambda^2 g'_S(\lambda, K)}{p(\lambda, a_i) - \lambda p'_S(\lambda, a_i)}, \quad i = 1, \dots, n. \quad (3.6)$$

Let us assume that

$$g'_K(S, K) \geq 0, \quad K > \lambda, \quad S > 0 \quad \text{and} \quad p''_{Sa}(S, a) > 0 \quad S > 0, \quad a > 0. \quad (3.7)$$

Note that from (3.6) we have $\frac{g(\lambda, K)}{p(\lambda, a_i)} + \frac{\lambda g'_S(\lambda, K)}{p(\lambda, a_i) - \lambda p'_S(\lambda, a_i)} = 0$ so, define

$$F(a, K) = \frac{g(\lambda, K)}{p(\lambda, a)} + \frac{\lambda g'_S(\lambda, K)}{p(\lambda, a) - \lambda p'_S(\lambda, a)}$$

and observe that

$$\frac{\partial F}{\partial K}(a, K) = \frac{g'_K(\lambda, K)}{p(\lambda, a)} + \frac{\lambda g''_{SK}(\lambda, K)}{p(\lambda, a) - \lambda p'_S(\lambda, a)} > 0 \quad (3.8)$$

in agreement with conditions (1.2) and (3.7). Thus, by the Implicit Function Theorem we can write

$$K = K(x_i, a_i) \quad \text{and} \quad \frac{\partial K}{\partial a_i} = - \frac{\frac{\partial F_i(x_i, a_i, K)}{\partial a_i}}{\frac{\partial F_i(x_i, a_i, K)}{\partial K}},$$

that is,

$$\frac{\partial K}{\partial a_i} = - \frac{g(\lambda, K) p'_a(\lambda, a)}{p(\lambda, a)^2} - \frac{\lambda g'_S(\lambda, K) (p'_a(\lambda, a) - \lambda p''_{Sa}(\lambda, a))}{(p(\lambda, a) - \lambda p'_S(\lambda, a))^2} > 0 \quad (3.9)$$

in accordance with conditions (1.2), (1.5), (1.8), and (3.7). Hence, by properties (3.8) and (3.9), K is an increasing function of a . Therefore, we have

$$\lambda < K_n < K_{n-1} < \dots < K_1 \quad (3.10)$$

furthermore, by conditions (1.3), (1.4), (3.5) and the continuity of $x_i^{H_K}$ and $x_i^{B_K}$, $i = 1, \dots, n$, for $\lambda < K < K_n$ the hyperplane (3.3) is above H_K and reaches H_K at, $x_n^{H_K} = x_n^{B_K}$ when $K = K_n$ (as K is increased both planes are displaced parallel). As K is increased further the hyperplane B_K cuts into H_K , reaches $x_{n-1}^{H_K} = x_{n-1}^{B_K}$ at $K = K_2$ and so on, reaches $x_1^{H_K} = x_1^{B_K}$ at $K = K_1$. After that, B_K cuts H_K such that now H_K is above B_K and condition (3.2) holds with an inverted inequality sign. In the process in that part of H_K which is already “above” B_K condition (3.2) holds with an inverted inequality sign. This means that the equilibria on this part of the plane have a two dimensional unstable manifold which fills a neighbourhood of this part of H_K . The points left behind by the intersection between H_K and B_K become destabilized. Farkas [3] called this phenomenon a *zip bifurcation*. We have thus arrived at

Theorem 1. Suppose that conditions (1.2) – (1.9), (1.11), and (3.7) hold and assume that $a_1 > a_2 > \dots > a_n$.

- (1) If $\lambda < K < K_n$, then each equilibrium in H_K is stable in the Liapunov sense and H_K is an attractor of system (1.1);
- (2) If $K_1 < K < \infty$, then all these equilibria are unstable and H_K is repellor;
- (3) If K is increased from one extreme to the other one of the interval (K_n, K_1) , then the hyperplane intersection of H_K and B_K is traveling through H_K from the vertex on the axis x_n to the vertex x_1 and the equilibria left behind get destabilized. For $K_n < K < K_1$ this hyperplane intersection divides H_K in two parts, “the upper one” is a repellor and “the lower one” is an attractor of the system, i. e., the system undergoes a zip bifurcation.

Proof. The assertion follows from the calculations carried out above. \square

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